1.3 Measuring and Recording Significant Data

Warm Up

Draw lines to match the following measured values, stated in SI units (International System of Units), with the appropriate everyday measurement.

SI measured value	Equivalent Everyday Measured Quantity
1 mm	width across a small fingernail
100 kPa	distance from your fingertips to your nose
1 L	pressure exerted by Earth's atmosphere
1 g	one-sixtieth of one minute in time
1 km	thickness of one dime
1 m	length of 10 football fields
1 s	volume of 1000 mL of milk
1 cm	mass of one raisin

SI Units

The **SI system (International System of Units)** is the modern metric system of measurement and the dominant system of international commerce and trade. The abbreviation SI comes from the French, Système international d'unités. The SI system was developed in 1960 and is maintained by the International Bureau of Weights and Measures (BIPM) in France. It is a non-static system devised around the seven base units listed below. Its non-static nature means SI units may be created and definitions may be modified through international agreement as measurement technology progresses and the precision of measuring devices improves. The SI system is the primary system of measurement in all but a small number of countries. The seven fundamental SI units are given in Table 1.3.1.

The following are some important SI rules or conventions:

- Unit symbols are always lower case letters unless the unit is named after a person. The one exception to this rule is L for litres. The full names of units are always written in lower case with the exception of *degrees Celsius*.
- Unit symbols should never be pluralized.
- Symbols should only be followed by a period at the end of a sentence.
- In general, the term *mass* replaces the term *weight*.
- The symbol cc should not be used in place of mL or cm^3.
- For values less than 1, use a 0 in front of the decimal point (e.g., 0.54 g not .54 g).
- A space rather than a comma should separate sets of three digits to the left *and right* of a decimal point. This practice is optional where there are only four digits involved.
- Use decimal fractions rather than common fractions (0.25 rather than ¼).
- Abbreviations such as sec, cc, or mps are avoided and only standard unit symbols such as s for second, cm^3 for cubic centimeter, and m/s for metre per second should be used.
- There is a space between the numerical value and unit symbol (4.6 g).
- Use the unit symbol in preference to words for units attached to numbers (e.g., 5.0 g/mol rather than 5.0 grams/mole). Note that 5.0 grams per mole is incorrect but five grams per mole is correct.
- A specific temperature and a temperature change both have units of degrees Celsius (°C).

Table 1.3.1 *The Fundamental SI Units*

Name	Unit Symbol	Quantity
metre	m	length
kilogram	kg	mass
second	s	time
ampere	A	electric current
kelvin	K	temperature
candela	cd	luminous intensity
mole	mol	amount of substance

Quick Check

Locate the SI error(s) in each of the following statements and correct them.

1. Ralph bought 6 kilos of potato salad. ______
2. The thickness of the oxide coating on the metal was 1/2 c.m. ______
3. The weight of 1 Ml of water is exactly 1 gms at 4 °c. ______
4. My teacher bought 9.0 litres of gasoline for her 883 c.c. motorcycle. ______
5. Rama's temperature increased by .9 C°. ______

Accuracy and Precision — The Quality of Measurements

Measured values like those listed in the Warm Up at the beginning of this section are determined using a variety of different measuring devices, some of which were included in section 1.1.

There are devices designed to measure all sorts of different quantities. The collection pictured in Figure 1.3.1 measures temperature, length, and volume. In addition, there are a variety of precisions (exactnesses) associated with different devices. The micrometer (also called a caliper) is more precise than the ruler while the burette and pipette are more precise than the graduated cylinder.

Despite the fact that some measuring devices are more precise than others, it is impossible to design a measuring device that gives perfectly exact measurements. All measuring devices have some degree of **uncertainty** associated with them.

The 1-kg mass kept in a helium-filled bell jar at the BIPM in Sèvres, France, is the only exact mass on the planet (Figure 1.3.2). All other masses are measured relative to this and therefore have some degree of associated uncertainty.

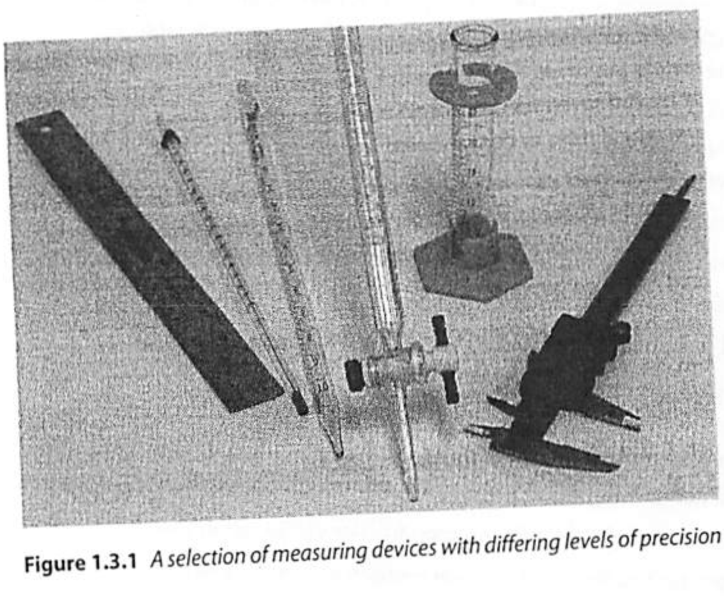


Figure 1.3.1 *A selection of measuring devices with differing levels of precision*

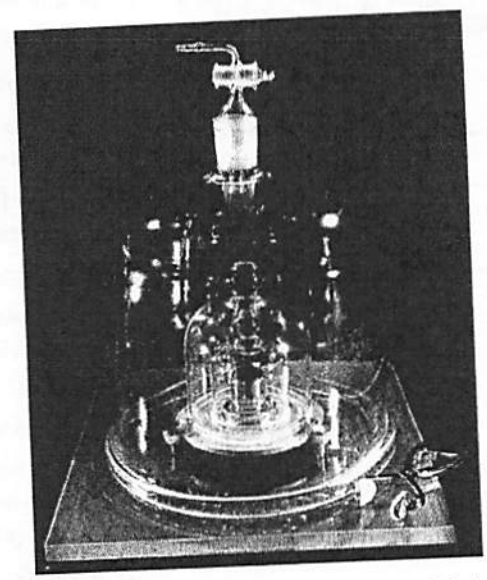


Figure 1.3.2 *This kilogram mass was made in the 1880s and accepted as the international prototype of the kilogram in 1889. (© BIPM — Reproduced with permission)*

Accuracy refers to the *agreement* of a particular value with the *true value.*

Accurate measurements depend on careful design and calibration to ensure a measuring device is in proper working order. The term **precision** can actually have two different meanings.

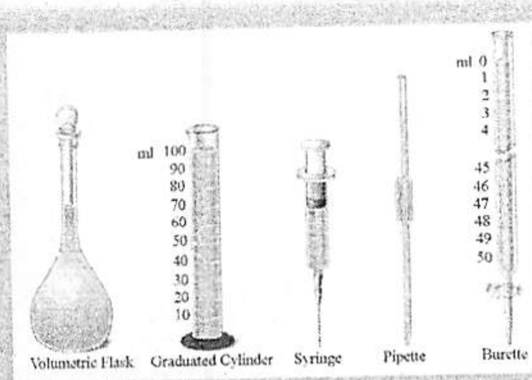
Precision refers to the reproducibility of a measurement (or the agreement among several measurements of the same quantity).

- or -

Precision refers to the exactness of a measurement. This relates to uncertainty: the lower the uncertainty of a measurement, the higher the precision.

Quick Check

Volumetric devices measure liquids with a wide variety of precisions.



1. Which of these is likely the *most precise*?

____

2. Which is likely the *least precise*?

____

3. Is the *most precise* device necessarily the *most accurate*?

____

____

4. Discuss your answers.

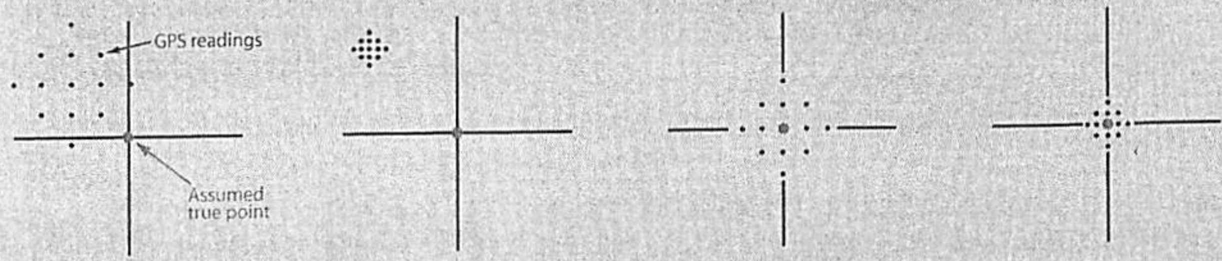
____

Types of Errors

No measurement can be completely precise. In fact, all measurements must have some degree of uncertainty associated with them, so it must be true that every measurement involves some degree of error. A group of measurements may tend to consistently show error in the same direction. Such a situation is called **systematic error**. When a group of errors occurs equally in high and low directions, it is called **random error**.

Quick Check

Four groups of Earth Science students use their global positioning systems (GPS) to do some geocaching. The diagrams below show the students' results relative to where the actual caches were located.



1. Comment on the precision of the students in each of the groups. (In this case, we are using the "reproducibility" definition of precision.)

____

2. What about the accuracy of each group?

____

3. Which groups were making systematic errors?

____

4. Which groups made errors that were more random?

____

Uncertainty

Every measurement has some degree of uncertainty associated with it. The uncertainty of a measuring device depends on its precision. The most precise measuring devices have the smallest uncertainties. The most common way to report the uncertainty of a measuring device is as a **range uncertainty.**

The **range uncertainty** is an *acceptable* range of values within which the true value of a measurement falls. This is commonly presented using the *plus* or *minus* notation (shown as +/–).

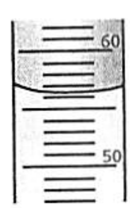


Figure 1.3.2 *A meniscus in a graduated cylinder*

An acceptable range uncertainty is usually considered plus or minus *one-tenth* to *one-half* the smallest division marked on a measuring scale. For the graduated cylinder in Figure 1.3.2, the smallest division marked on its scale is 1 mL. One-half of this division is 0.5 mL. Therefore, reading the volume from the bottom of the meniscus would give a value of 56.3 mL +/– 0.5 mL. Because it is possible to go down to one-tenth of the smallest division on the scale as a range, the volume could also be reported as 56.3 mL +/– 0.1 mL. Notice that the unit is given twice, before and after the +/– symbol. This is the correct SI convention.

When *two uncertain values* with range uncertainties are *added or subtracted*, their *uncertainties must be added.*

Quick Check

1. Sum the following values:

   27.6 mL +/– 0.2 mL
   + 14.8 mL +/– 0.2 mL

2. Subtract the following values:

   19.8 mL +/– 0.2 mL
   – 7.2 mL +/– 0.2 mL

Compare your answers with your partner. Explain to your partner why the answer to question 1 makes sense. Have your partner explain the answer to question 2.

Absolute uncertainty refers to exactly how much higher or lower a measured value is than an accepted value.

In such a case, an accepted value is the value considered to be the best measurement available. Constants such as the speed of light or the boiling point of water at sea level are accepted values. For absolute uncertainties, a sign should be applied to indicate whether the measured value is above or below the accepted value.

Another common way to indicate an error of measurement is as a percentage of what the value should be.

$$\textbf{percentage error} = \frac{\text{(measured value} - \text{accepted value)}}{\text{accepted value}} \times 100\%$$

Quick Check

A student weighs a Canadian penny and finds the mass is 2.57 g. Data from the Canadian Mint indicates a penny from that year should weigh 2.46 g.

1. What is the absolute uncertainty of the penny's mass?

2. What is the percentage error of the penny's mass?

3. Suggest a reasonable source of the error.

   ____

Significant Figures

When determining the correct value indicated by a measuring device, you should always record all of the **significant figures** (sometimes called "sig figs").

> The number of significant figures in a measurement includes *all* of the certain figures *plus* the *first* uncertain (estimated) figure.

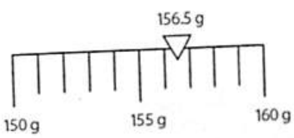


Figure 1.3.3 *Scale on a measuring device*

Say the scale on a measuring device reads as shown in Figure 1.3.3. (Note that *scale* may refer to the numerical increments on any measuring device.) The measured value has *three certain figures* (1, 5, and 6) and *one uncertain* figure (5 or is it 4 or 6?). Hence the measurement has *four significant figures.*

The following rules can be applied to determine how many figures are significant in any measurement.

Counting Significant Figures in a Measured Value

1. All non-zero digits are significant.
2. All zeros between non-zero digits are significant. Such zeros may be called *sandwiched* or *captive* zeros.
3. Leading zeros (zeros to the left of a non-zero digit) are *never* significant.
4. Trailing zeros (zeros to the right of a non-zero digit) are *only* significant if there is a *decimal* in the number.

Another way to determine the number of significant figures in a measured value is to simply express the number in scientific notation and count the digits. This method nicely eliminates the non-significant leading zeros. However, it is only successful if you recognize when to include the trailing (right side) zeros. Remember that trailing zeros are only significant if there is a decimal in the number. If the trailing zeros are significant, they need to be included when the number is written in scientific notation.

Sample Problems — Counting Significant Figures in a Measured Value

Determine the number of significant figures in each example.

1. 0.09204 g
2. 87.050 L

What to Think about	How to Do It
Question 1	
1. To begin with, apply rule 3: leading zeros are never significant. Note that the position of the zero relative to the decimal is irrelevant. These are sometimes referred to as *place holding* zeros. The underlined leading zeros are not significant.	0.09204 g
2. Apply rule 1 next: all non-zero digits are significant. The underlined digits are significant.	0.09204 g
3. Finally, apply rule 2: the captive zero is significant. The underlined zero is sandwiched between two non-zero digits so it is significant.	0.09204 g
4. A check of the number in scientific notation, 9.204×10^{-2} g, also shows four significant figures.	0.09204 g has four significant figures.

Continued

Sample Problems — *Continued*

Question 2

1. Apply rule 1: all non-zero digits are significant. All of the underlined digits are significant.	87.050 L
2. Apply rule 2: the captive zero is significant. The underlined zero is between two non-zero digits so it is significant.	87.050 L
3. Finally, consider the trailing or right-side zero. Rule four states that such zeros are only significant if a decimal is present in the number. Note that the position of the zero relative to the decimal is irrelevant.	
As this number does contain a decimal, the underlined trailing zero is significant.	87.050 L
4. Check: 8.7050×10^1 L has five sig figs (note that the right-side zero is retained).	87.050 L has **five** significant figures.

Practice Problems — Counting Significant Figures in a Measured Value

1. How many significant figures are in each of the following measured values?

   (a) 425 mL __________

   (b) 590.50 g __________

   (c) 0.00750 s __________

   (d) 1.50×10^4 L __________

   (e) 3400 m __________

2. Round the following measurements to the stated number of significant figures.

   (a) 30.54 s (3 sig figs) __________

   (b) 0.2895 g (3 sig figs) __________

   (c) 4.49 m (2 sig figs) __________

   (d) 100.4°C (2 sig figs) __________

Significant Figures and Arithmetic — Multiplication and Division

The answer to a multiplication or division problem should have only as many figures as the number having the *least significant digits* in the problem.

Sample Problems — Significant Figures in Multiplication and Division Calculations

Give the answer to each of the following problems with the correct number of significant figures:

1. 8.2 m × 9.47 m =
2. 12 970.0 g ÷ 530.8 mL =

What to Think about	How to Do It
Question 1 1. Begin by applying rules 1 to 4 to determine the number of significant figures contained in each number in the problem.	8.2 m × 9.47 m = 77.654 m^2 2 sig figs) (3 sig figs)
2. Express the answer should be expressed to *two* significant figures.	77.654 rounds to 78 m^2
Question 2 1. The second example involves more difficult numbers. Apply rules 1 to 4 to quickly determine the number of sig figs in each value.	12 970.0 g ÷ 530.8 mL = 24.434815... g/mL (6 sig figs) (4 sig figs)
2. Express the answer to *four* significant figures.	24.434815... rounds to 24.43 g/mL

Quick Check

Give the answer to each of the following problems with the appropriate unit and the correct number of significant figures:

1. 0.14 m × 14.00 m = ____________________
2. 940 g ÷ 0.850 mL = ____________________
3. 0.054 g ÷ 1.10 s = ____________________

Significant Figures and Arithmetic – Addition and Subtraction

The answer to an addition or subtraction problem should have only as many figures as the *least precise* (least exact) number in the problem. The number of significant figures in the answer is determined by considering the *place value* of the last significant figure in each number in the problem. (If both measurements include numbers to the right of a decimal, this means you simply round to the smallest number of decimal places.)

Sample Problems — Significant Figures in Addition and Subtraction Calculations

Give the answer to each of the following problems with the correct number of significant figures:

1. 246.812 cm + 1.3 cm =
2. 25 510 km − 7 000 km =

What to Think about	How to Do It
Question 1 1. In addition and subtraction problems, the most important thing is to determine the *place value* of the last significant figure in each number. 246.812 contains non-zero digits only, so the last significant figure is the last 2, which occupies the thousandths place. The 3 in 1.3 is the last significant figure and occupies the tenths place.	246.812 + 1.3 248.112 cm
2. When these two place values are compared, the tenths is *less precise*; that is, it is *less exact* than the thousandths place. Round the final answer to the tenths place, resulting in a number with **four** significant figures. Notice that this rounded to one decimal place.	248.112 rounds to 248.1 cm
Question 2 1. The problem involves adding and subtracting. Determine the place value of the last significant figure in each number. The last significant figure in 25 510 is the 1. It is in the tens place. The 7 is the last significant figure in 7000. It is in the thousands place.	25 510 − 7 000 18 510 km
2. Thousands are far less exact than tens so round the final answer to the thousands place resulting in an answer with two significant figures. Notice that the decimal place simplification does not apply in this example.	18 510 km rounds to → 19 000 km

Practice Problems— Significant Figures in Addition and Subtraction Calculations

Give the answer to each of the following problems with the correct number of significant figures:

1. 16.407 mL + 5.70 mL = ____________________
2. 0.32 g + 0.009 g = ____________________
3. 750 m + 8.001 m = ____________________

Significant Figures — Summary

- When there are a series of arithmetic functions to be performed with measurements, always remember to apply the correct order of operations.
- Use the proper rules for determining the number of significant figures *in each step.*
- Remember that significant figures apply to measured values only.
- They do *not* apply to counted values or values that are defined.

Quick Check

Answer each of the following with the correct number of significant figures.

1. $\frac{9.825\text{g} - 9.804\text{ g}}{9.825\text{ g}} \times 100\% = \frac{0.021\text{ g}}{9.825\text{ g}} \times 100\% =$
2. 804.08 g ÷ (424.4 mL + 42.8 mL) =
3. (3.202 m × 4.80 m) / (26.4 min – 17.3 min) =
4. 7.0×10^2 s + 6.010×10^3 s =

1.3 Activity: Connecting Significant Figures with Uncertainty

Question
What is the uncertainty in an area calculation?

Background
A student was assigned the task of determining the area of a small Post-It® note. She was instructed to use a low-precision plastic ruler, marked in increments of centimetres. Knowing it is acceptable to use between one-tenth and one-half the smallest increments on the ruler, the student decided on a range uncertainty of +/– 0.2 cm.

Dimensions	Measurements
Height	4.6 cm +/– 0.2 cm
Width	5.5 cm +/– 0.2 cm

Procedure
1. Calculate the smallest possible area of the Post-It® (keep **all digits**).

2. Calculate the largest possible area of the Post-It® (again, keep all digits).

3. Determine the area as the average of the values calculated in one and two (keep all figures in your calculation).

Results and Discussion
1. Determine an uncertainty that will include the smallest and largest possible areas (note that uncertainty may be expressed *in one place value only).*

2. The uncertain place will determine what place your area should be rounded to. Now express the area of the Post-It™ including the range uncertainty.

3. How does the number of significant figures in your answer compare with what you would have expected based on what you've learned about calculating with significant figures in this section?

1.3 Review Questions

1. Determine the errors and correct them according to the SI system:

   (a) The package contained 750 Gm of linguini.

   (b) The car accelerated to 90 km per hour in 10 sec.

   (c) The recipe called for 1 ML of vanilla and 250 cc of milk.

   (d) Jordan put 250 gms of mushrooms on his 12 inch pizza.

2. A zinc slug comes from a science supply company with a stated mass of 5.000 g. A student weighs the slug three times, collecting the following values: 4.891 g, 4.901 g, and 4.890 g. Are the student's values accurate? Are they precise (consider both meanings)?

3. A student doing experimental work finds the density of a liquid to be 0.1679 g/cm^3. The known density of the liquid is 0.1733 g/cm^3. What is the absolute error of the student's work? What is the percent error?

4. Two students weigh the same object with a known mass of 0.68 g. One student obtains a mass of 0.72 g, while the other gets a mass of 0.64 g. How do their percent errors compare? How do their absolute errors compare?

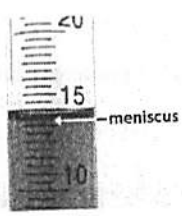
5. In an experiment to determine the density of a liquid, a maximum error of 5.00% is permitted. If the true density is 1.44 g/cm^3, what are the maximum and minimum values within which a student's answer may fall into the acceptable range?

6. What is the mass, including uncertainty, arrived at as the result of summing 45.04 g +/– 0.03 g, and 39.04 g +/– 0.02 g?

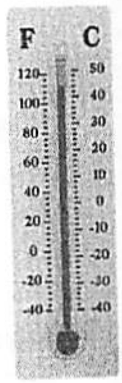
7. What is the smallest number that could result from subtracting 22 m +/– 2 m from 38 m +/– 3 m?

8. The dimensions of a rectangle are measured to be 19.9 cm +/– 0.1 cm and 2.4 +/– 0.1 cm. What is the area of the rectangle, including the range uncertainty?

9. Read each of the following devices, including a reasonable range uncertainty:



(a)



(b)

10. Determine the number of significant figures in each of the following measurements:

    (a) 0.1407 m ______________

    (b) 21.05 mg ______________

    (c) 570.00 km ______________

    (d) 0.0030 cm ______________

    (e) 250 m ______________

    (f) 10 035.00 cm^3 ______________

    (g) 2800 g ______________

    (h) 5000°C ______________

    (i) 1.1×10^2 kPa ______________

    (j) 5.35×10^{-2} m/h ______________

11. Express the following in proper form scientific notation. Then indicate the correct number of significant figures in the value.

(a) 4907 L ______________________

(b) 0.000 052 m ______________________

(c) 7900 g ______________________

(d) 0.060 30 ft ______________________

(e) 790.0 lb ______________________

12. Carry out the following operations and give the answers with the correct number of significant figures. Pay close attention to the units.

(a) 14.6 cm × 12.2 cm × 9.3 cm

(b) 28.0 m × 16.0 m × 7.0 m

13. A chunk of nickel has a mass of 9.0 g and a volume of 1.01 mL. What is its density?

14. The density of copper is 8.9 g/mL. What is the mass of a 10.8 mL piece of copper?

15. Carry out the following operations and give the answer with the correct number of significant figures.

(a) 608 g + 7 g + 0.05 g

(b) 481.33 mL − 37.1 mL

(c) 6620 s + 35.7 s + 1.00 s

(d) 0.007 m + 0.100 m + 0.020 m

16. Determine the answer with the correct number of significant figures:

$$\frac{1.415\ \text{g}}{1.6\ \text{mL}} + \frac{0.240\ \text{g}}{0.311\ \text{mL}} + \frac{40.304\ \text{g}}{0.2113\ \text{mL}}$$

17. Determine the answer to each the following with the correct number of significant figures:

(a) $\dfrac{8.4\ \text{g} + 3.0\ \text{g} + 4.175\ \text{g}}{3}$

(b) $\dfrac{9.00 \times 10^{-23}\ \text{units} \times 2.9900 \times 10^{-25}\ \text{units}}{2.9 \times 10^{-9}\ \text{units}}$

(c) $\dfrac{(5.9 \times 10^{-12}\ \text{u} + 7.80 \times 10^{-13}\ \text{u})}{(4 \times 10^{12}\ \text{u} + 6.700 \times 10^{13}\ \text{u})}$

18. The label on a bottle of mood-elevating medication states that each tablet contains 25.0 mg of imipramine. A test conducted by the bureau of standards shows a tablet to contain 28.0 mg. Legally, drug companies are allowed to be within plus or minus 5% of their labelled quantities.

(a) Give the *percentage* uncertainty for the imipramine tablets:

(b) Is the drug company within the legally allowed limits for their tablets?